

Onsager's Variational Principle in Soft Matter Dynamics

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In soft matter, many equations have been proposed to describe the complex behavior of the system in non-equilibrium state. Examples are (i) the phase separation kinetics (model H) : the coupled equations for the component diffusion and the fluid flow, (ii) the gel dynamics: the coupled equation for the permeation of solvent and the deformation of gel, (iii) nemato-hydrodynamics: the equations describing the director field and the velocity field, and (iv) kinetic equation for electrolyte solutions which describe the time evolution of ion-concentration, electric field, and solvent flow etc. These equations were originally proposed as a phenomenological equations or a generalization of the existing equations, have been tested, and are now forming a common base the soft matter physics. Here I would like to show that these equations can be derived from a variational principle which I shall call Onsager's variational principle.

The variational principle is a simple rewriting of the conventional thermodynamics of irreversible processes. For the iso-thermal system, the phenomenological time evolution equation can be written as

$$\frac{dX_i}{dt} = -\mu_{ij} \frac{\partial A}{\partial X_j} \quad (1)$$

Where X_i is the set of the variable describing the non-equilibrium state, $A(X)$ is the free energy of the system and $\mu_{ij}(X)$ is the kinetic coefficients. Due to Onsager's reciprocal relation $\mu_{ij} = \mu_{ji}$ equation(1) can be stated in the following variational principle. The time evolution of the system is determined by \dot{X}_i which minimize the following quadratic function

$$R = \frac{1}{2} \sum \zeta_{ij} \dot{X}_i \dot{X}_j + \sum \frac{\partial A}{\partial X_i} \dot{X}_i \quad (2)$$

where ζ_{ij} is the inverse matrix of μ_{ij} .

Here I will show (i) the above equations for soft matter dynamics can be derived from this variational principle, (ii) the variational principle is convenient since it allows a great flexibility in choosing the state variable and guarantees the Onsager's reciprocal relation in the final equation.

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